Gaia Astrometric Accuracy

Now and 100 years ago

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Outline

- Gaia Accuracy at mean epoch
- Propagation model
  - position
  - covariances
- Gaia accuracy in the last century
- The radial velocity issue
- Conclusions/discussion

G A I A

- 10^9 stars
- 25 μas @ V = 15 mag

ESA mission
Launch: 2013
Mission : 5 yrs

- Photometry (~ 25 bands)
- Radial velocity
- Low resolution spectroscopy

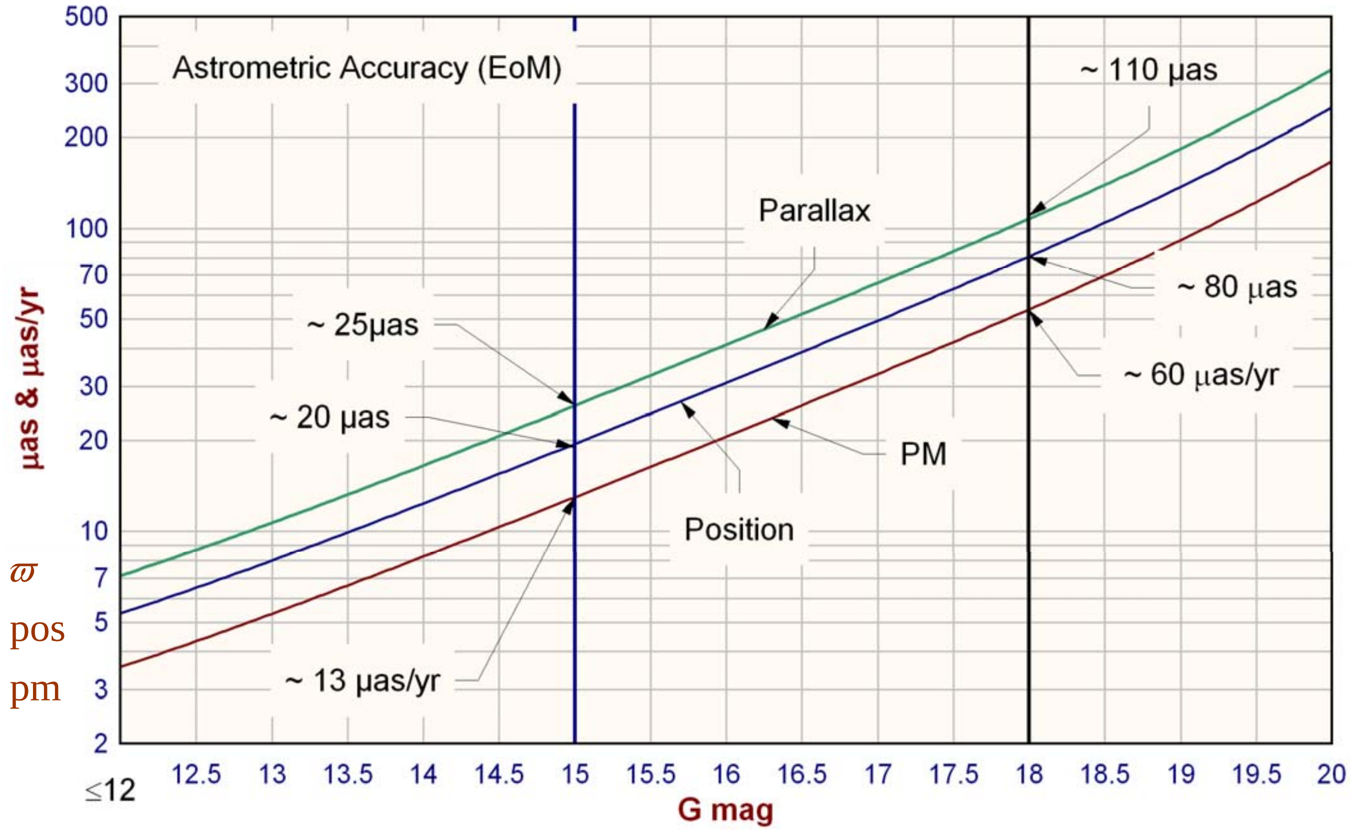
Mission requirements summary

- *A Stereoscopic Census of Our Galaxy*
- Astrometry (V < 20):
  - completeness to 20 mag (on-board detection) 10^9 stars
  - parallax accuracy: 7 μas at <10 mag; 12–25 μas at 15 mag 100–300 μas at 20 mag
- Photometry (V < 20):
  - astrophysical diagnostics (low-dispersion photometry) + chromaticity
  - 8–20 mmag at 15 mag: Teff ~ 200 K, log g, [Fe/H] to 0.2 dex, extinction
- Radial velocity (V < 16.5–17):
  - Third component of space motion, perspective acceleration
  - <1 km/s at 13-13.5 mag and <15 km/s at 16.5-17 mag

Gaia Accuracy at mean epoch

Gaia DPAC

- Five year mission, sky –averaged
  - reference value: $\sigma_{\varpi} = 25\mu as$ for $G = 15$
  - based on data from J. De Bruijne (ESA)



NAROO-Gaia, 21 June 2012

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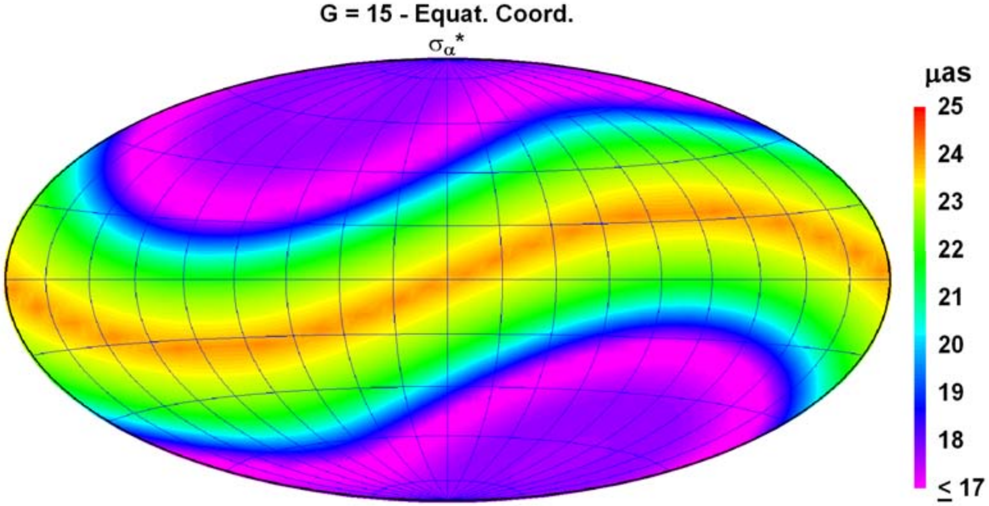
Observatoire de la Côte d'Azur

Sky distribution – Positions (~ 2016.5)

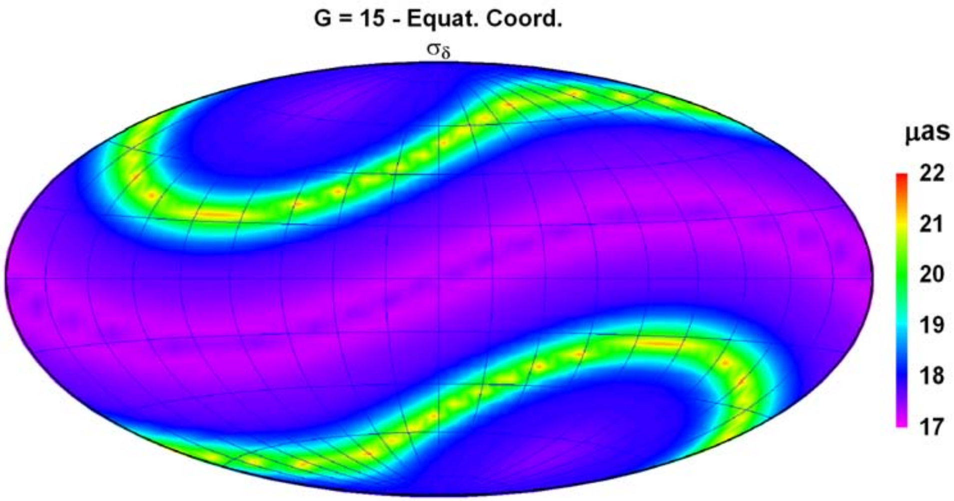
- Plots for G =15, but scalable to other magnitudes



$\sigma_\alpha^* - \mu\mathrm{as}$

$$<\sigma_\alpha^*> = 21\mu\mathrm{as}$$

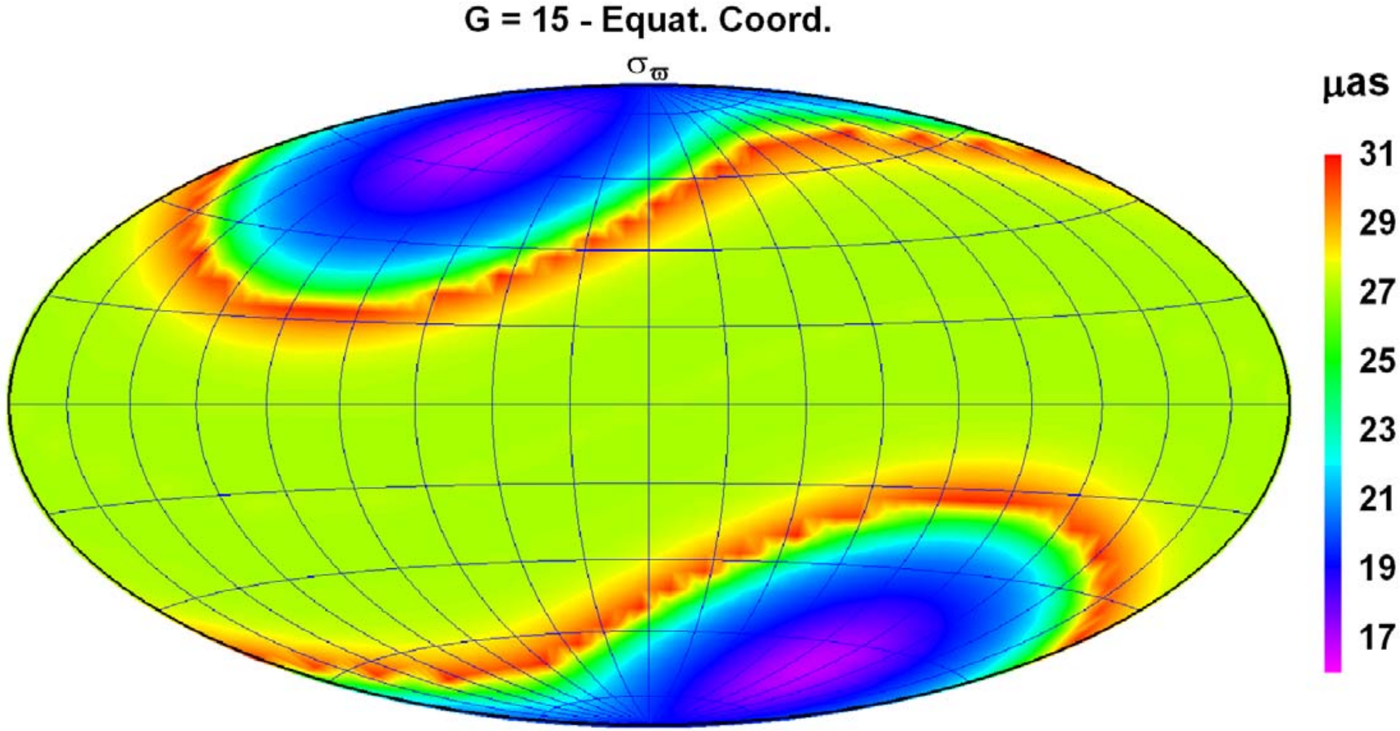
$$<\sigma_\delta> = 18\mu\mathrm{as}$$



$\sigma_\delta - \mu\mathrm{as}$

Sky distribution – Parallaxes

- Plot for G =15, but scalable to other magnitudes

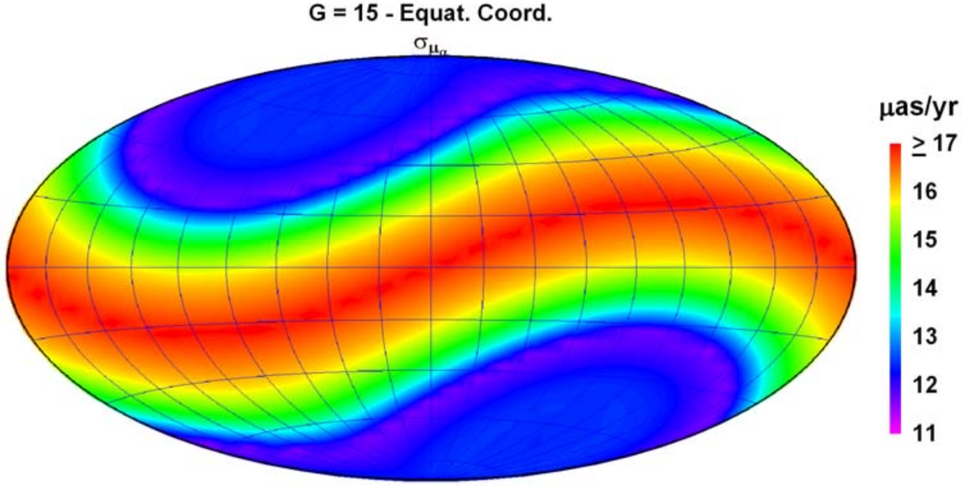


$\sigma_{\varpi} - \mu\text{as}$

$<\sigma_{\varpi}> = 25\mu\text{as}$

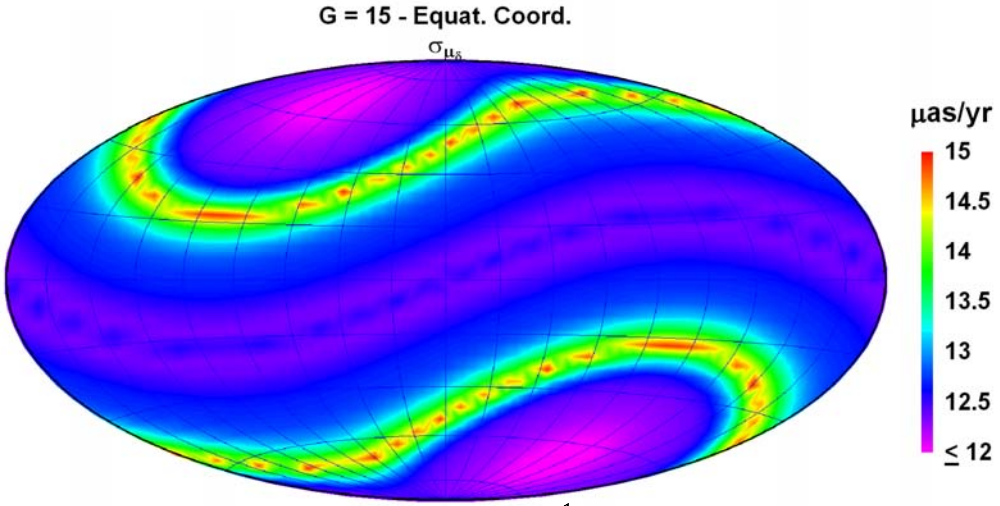
Sky distribution – Proper motions

- Plots for G =15, but scalable to other magnitudes



$\sigma^*_{\mu_\alpha}$ – μas yr^{-1}

$$<\sigma^*_{\mu_\alpha}> = 15\mu\text{as yr}^{-1}$$



σ_{μ_δ} – μas yr^{-1}

$$<\sigma_{\mu_\delta}> = 13\mu\text{as yr}^{-1}$$

Gaia DPAC

Astrometric Propagation

Mathematical modelling

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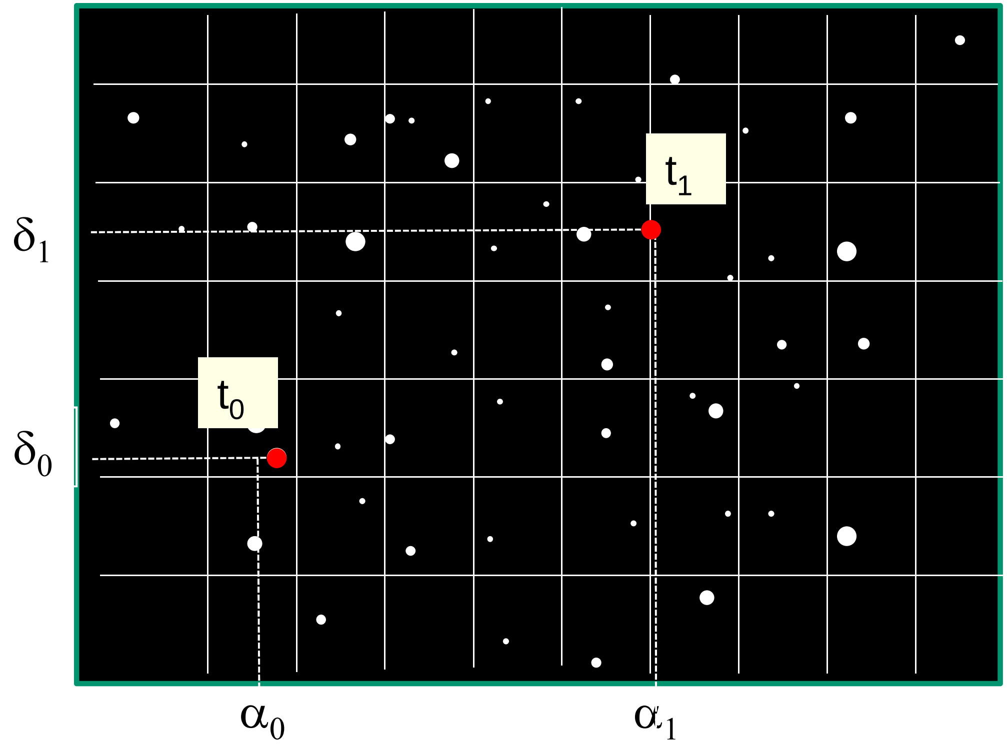
Position propagation: first order



$$(\alpha_1 - \alpha_0)\cos\delta = \mu_\alpha (t_1 - t_0)$$

$$(\delta_1 - \delta_0) = \mu_\delta (t_1 - t_0)$$

insufficient accuracy with the Gaia data

Proper motion: first order definition

$$(\alpha_1 - \alpha_0)\cos\delta = \mu_\alpha (t_1 - t_0)$$

$$(\delta_1 - \delta_0) = \mu_\delta (t_1 - t_0)$$

$\Longrightarrow$ First terms in an expansion in power of t

★ V = 50 km/s @ 10 pc

Terms in	amplitude	Δα*, Δδ over 50 yrs	Δα*, Δδ over 100 yrs
t	1″/yr	50″	100″
t^2	5 μas/yr^2	12 mas	50 mas
t^3	25x10^{-6} μas/yr^3	3 μas	25 μas

Proper motion: accurate definition

- Components of the angular velocity on the tangent plane
  - this is a time derivative at a reference epoch

$$\boxed{\boldsymbol{\mu} = \frac{\mathbf{u}\times(\mathbf{V}\times\mathbf{u})}{r} = \frac{\mathbf{V}_\perp}{r}}$$

- Time dependant definition even with constant 3D velocity
  - with t_0 and t_1 given, one must select the epoch at which the PM is referred to and propagate the position
  - Assumption: the 3D motion is rectilinear and uniform
  - the projected motion is not uniform. PM components are not constant
- Not so trivially related to the first differences $\Delta\alpha^*$ and $\Delta\delta$
  - The angular coordinates have non-zero time derivatives > first order

Single star propagation model - I

position vector at t

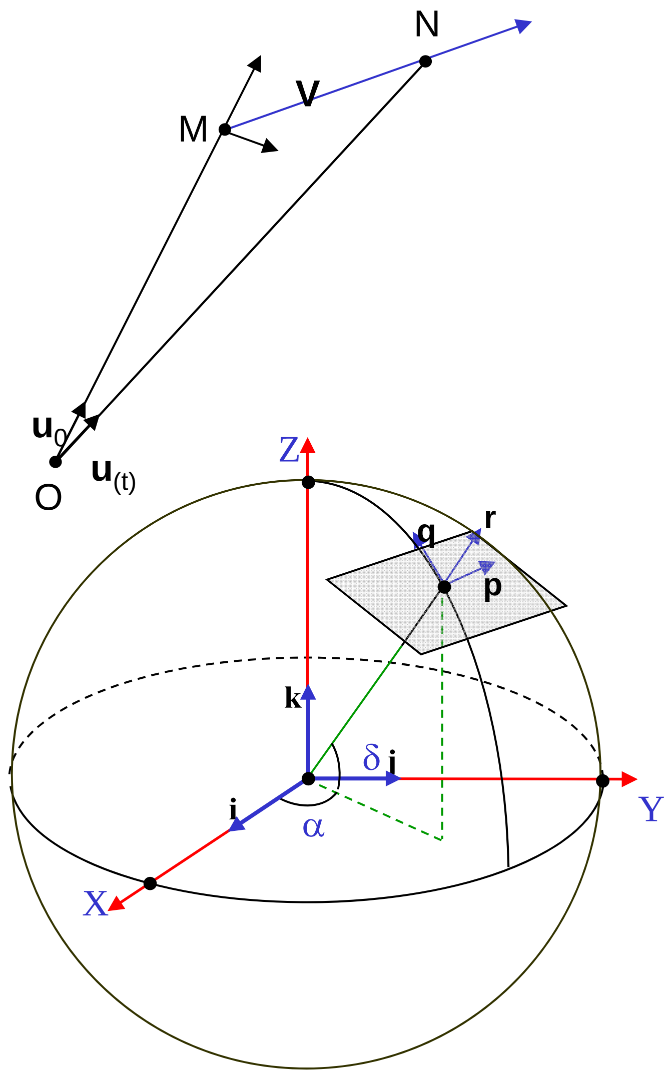
$$\mathbf{r}(t) = \mathbf{r}_0 + \mathbf{v}(t - t_0)$$

components of proper motion at t_0

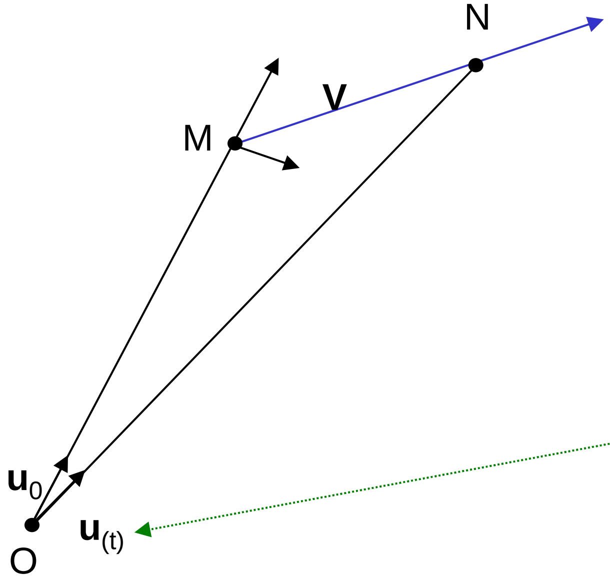
$$\mu_r = \frac{v_r}{r} \qquad \mu_\alpha = \frac{\mathbf{V}.\mathbf{p}}{r} \qquad \mu_\delta = \frac{\mathbf{V}.\mathbf{q}}{r}$$

angular velocity

$$\mathbf{W} = \frac{\mathbf{V}}{r} \qquad \mathbf{W} = \mathbf{W}_\perp + W_r\,\mathbf{u}_0$$



Single star propagation model- II



unit vector at t:

radial velocity

$$\mathbf{u}(t) = \mathbf{u}_0 + \mathbf{W}_\perp\, t - \left[W_\perp^2 \mathbf{u}_0 + 2\boxed{W_r}\,\mathbf{W}_\perp\right]\frac{t^2}{2} + \left[2\,W_r\,W_\perp^2 \mathbf{u}_0 + \left(2W_r^2 - W_\perp^2\right)\mathbf{W}_\perp\right]\frac{t^3}{2}$$

Propagation model to 3rd order

- Projection of $\mathbf{u}(t) - \mathbf{u}_0$ on the tangent plane
  - parallel projection and not the central one

$$
\begin{aligned}
\Delta\alpha\cos\delta_0 = {} & \mu_\alpha\, t \\
& - \left[\mu_r\,\mu_\alpha - \tan\delta_0\,\mu_\alpha\,\mu_\delta\right] t^2 \\
& + \left[\mu_r^2\,\mu_\alpha - 2\tan\delta_0\,\mu_r\,\mu_\alpha\,\mu_\delta + \tan^2\delta\,\mu_\alpha\,\mu_\delta^2 - \frac{\mu_\alpha^3}{3\cos^2\delta_0}\right] t^3
\end{aligned}
$$

$$
\begin{aligned}
\Delta\delta = {} & \mu_\delta\, t \\
& - \left[\mu_r\,\mu_\delta + \frac{\tan\delta_0}{2}\,\mu_\alpha^2\right] t^2 \\
& + \left[\mu_r^2\,\mu_\delta + \tan\delta_0\,\mu_r\,\mu_\alpha^2 - \frac{\mu_\alpha^2\,\mu_\delta}{2\cos^2\delta_0} - \frac{\mu_\delta^3}{3}\right] t^3
\end{aligned}
$$

Proper motion and first differences

- inversion to solve for the proper motion components

$$\begin{aligned}\mu_\alpha\, t &= \Delta\alpha^\star \\ &\quad + \Delta\alpha^\star\, \mu_r\, t - \tan\delta_0\, \Delta\alpha^\star\, \Delta\delta \\ &\quad + \frac{3\cos^2\delta_0 - 1}{6\,\cos^2\delta_0}\,(\Delta\alpha^\star)^3 - \tan\delta_0\,\Delta\alpha^\star\,\Delta\delta\,\mu_r t\end{aligned}$$

$$\begin{aligned}\mu_\delta\, t &= \Delta\delta \\ &\quad + \Delta\delta\,\mu_r\, t + \frac{1}{2}\tan\delta_0\,(\Delta\alpha^\star)^2 \\ &\quad + \frac{2\cos^2\delta_0 - 1}{2\,\cos^2\delta_0}\,(\Delta\alpha^\star)^2\,\Delta\delta + \frac{1}{2}\tan\delta_0\,(\Delta\alpha^\star)^2\,\mu_r t + \frac{\Delta\delta^3}{3}\end{aligned}$$

Covariance Matrix I

- For each star, Gaia delivers the 5 astrometric parameters and their covariance matrix

$$
C(t_0) = \begin{array}{c c} & \begin{array}{ccccc} \alpha^* & \delta & \varpi & \mu_\alpha & \mu_\delta \end{array} \\ \begin{array}{c} \alpha^* \\ \delta \\ \varpi \\ \mu_\alpha \\ \mu_\delta \end{array} & \left[\begin{array}{ccccc} \\ \\ & & C_{ij} & & \\ \\ \\ \end{array} \right] \end{array}
\qquad c_{ii} = \sigma^2(p_i)\ ;\ \ c_{11} = \sigma^2(\alpha^*),\ \ldots\text{etc}
$$

- The matrix is not diagonal even at Gaia epoch
  - correlations are relatively small (~ < 0.3)
- To get the accuracy at any other epoch, one must compute $C(t)$
  - first approximation

$$
\boxed{\alpha(t) = \alpha_0 + \frac{\mu_\alpha}{\cos\delta}(t - t_0) \quad \Rightarrow \quad \sigma^2_{\alpha*} \approx \sigma^2_{\alpha_0*} + \sigma^2_{\mu_\alpha}(t - t_0)^2}
$$

Covariance Matrix II

- General case use of the full astrometric model

$$\begin{aligned}
\alpha(t) &= F_\alpha(\alpha_0, \delta_0, \mu_\alpha^0, \mu_\delta^0, \mu_r, t) \\
\delta(t) &= F_\delta(\alpha_0, \delta_0, \mu_\alpha^0, \mu_\delta^0, \mu_r, t) \\
\mu_\alpha(t) &= F_{\mu_\alpha}(\alpha_0, \delta_0, \mu_\alpha^0, \mu_\delta^0, \mu_r, t) \\
\mu_\delta(t) &= F_{\mu_\delta}(\alpha_0, \delta_0, \mu_\alpha^0, \mu_\delta^0, \mu_r, t)
\end{aligned}
\quad \longrightarrow \quad
J(t) = \begin{array}{c|c} & \alpha^* \quad \delta \quad \mu_\alpha \quad \mu_\delta \\ \begin{array}{c} \alpha^* \\ \delta \\ \mu_\alpha \\ \mu_\delta \end{array} & \left[\begin{array}{c} \\ \dfrac{\partial F_i}{\partial p_j} \\ \\ \end{array} \right] \end{array}$$

$$\boxed{C(t) = J C(t_0) J^T}$$

- Both the positional and covariance propagations have been used in the following illustrations

Application to Gaia

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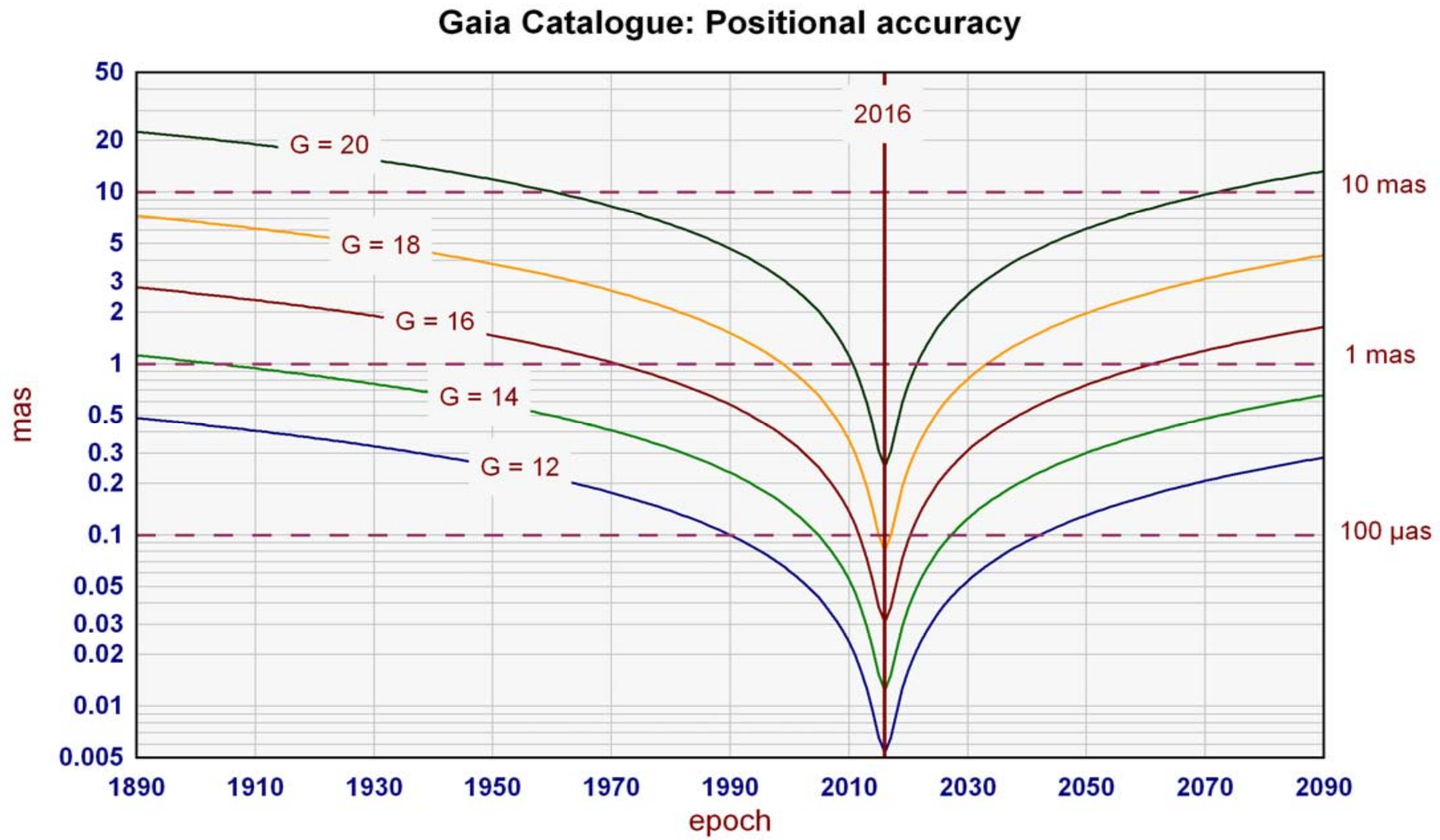
Gaia Accuracy in the past and future

- Covariance matrix fully propagated at t = 1890..2090 step 1 yr
  - sky averaged accuracy
  - mean accuracy between α and δ



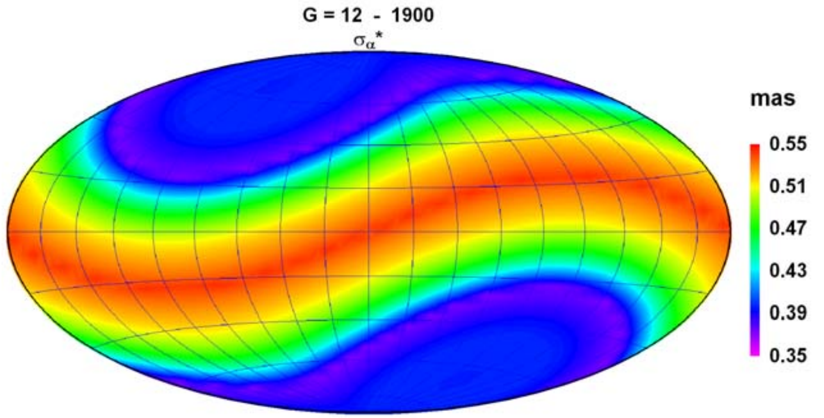
Detailed plots

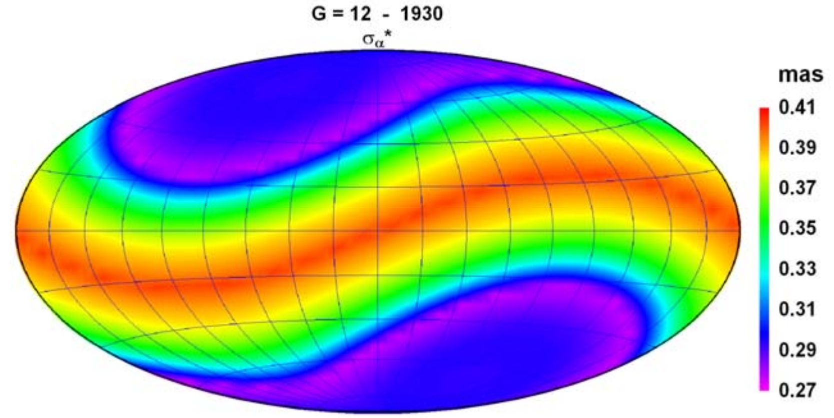
- Grid of computation
  - Two magnitudes
    - $G = 12$ - $G = 15$
  - Four epochs
    - 1900, 1930, 1960, 1990
  - Two coordinates
    - α, δ
- Sky distribution in equatorial coordinates

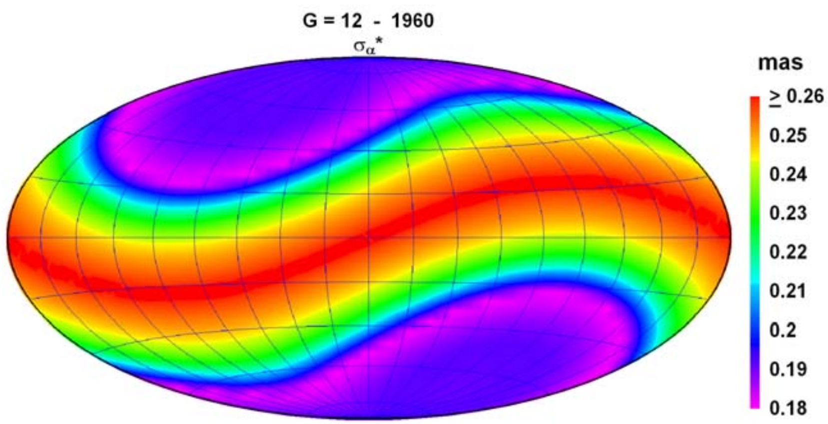
$$\sigma_\alpha^* = \sigma_\alpha \cos\delta \quad - \quad \sigma_\delta$$

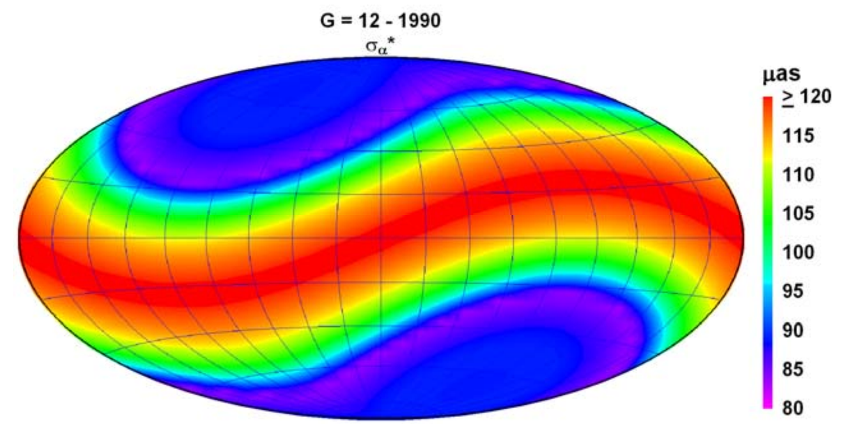
  - units: mas or μas

σ_α 1900-1990 - G = 12

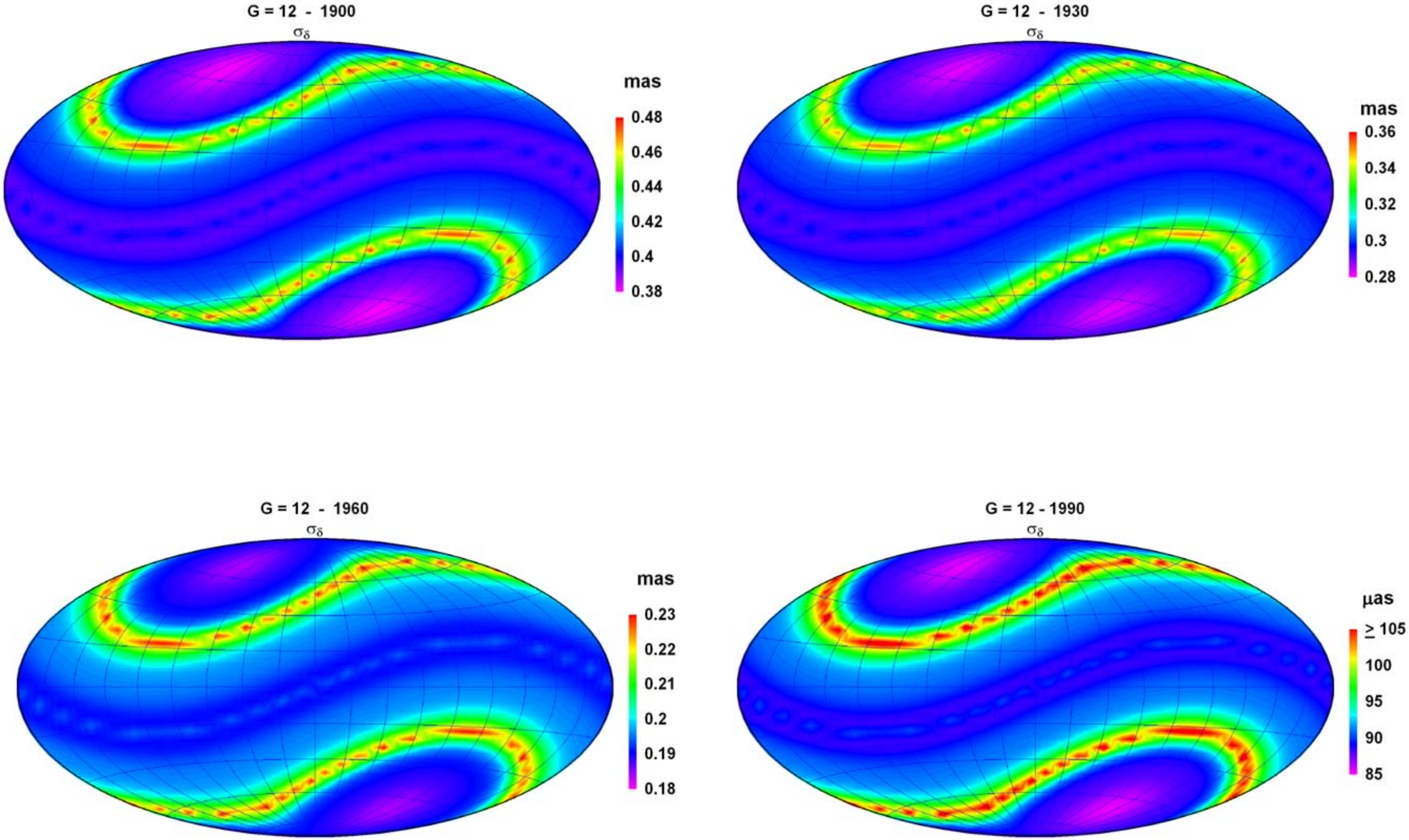




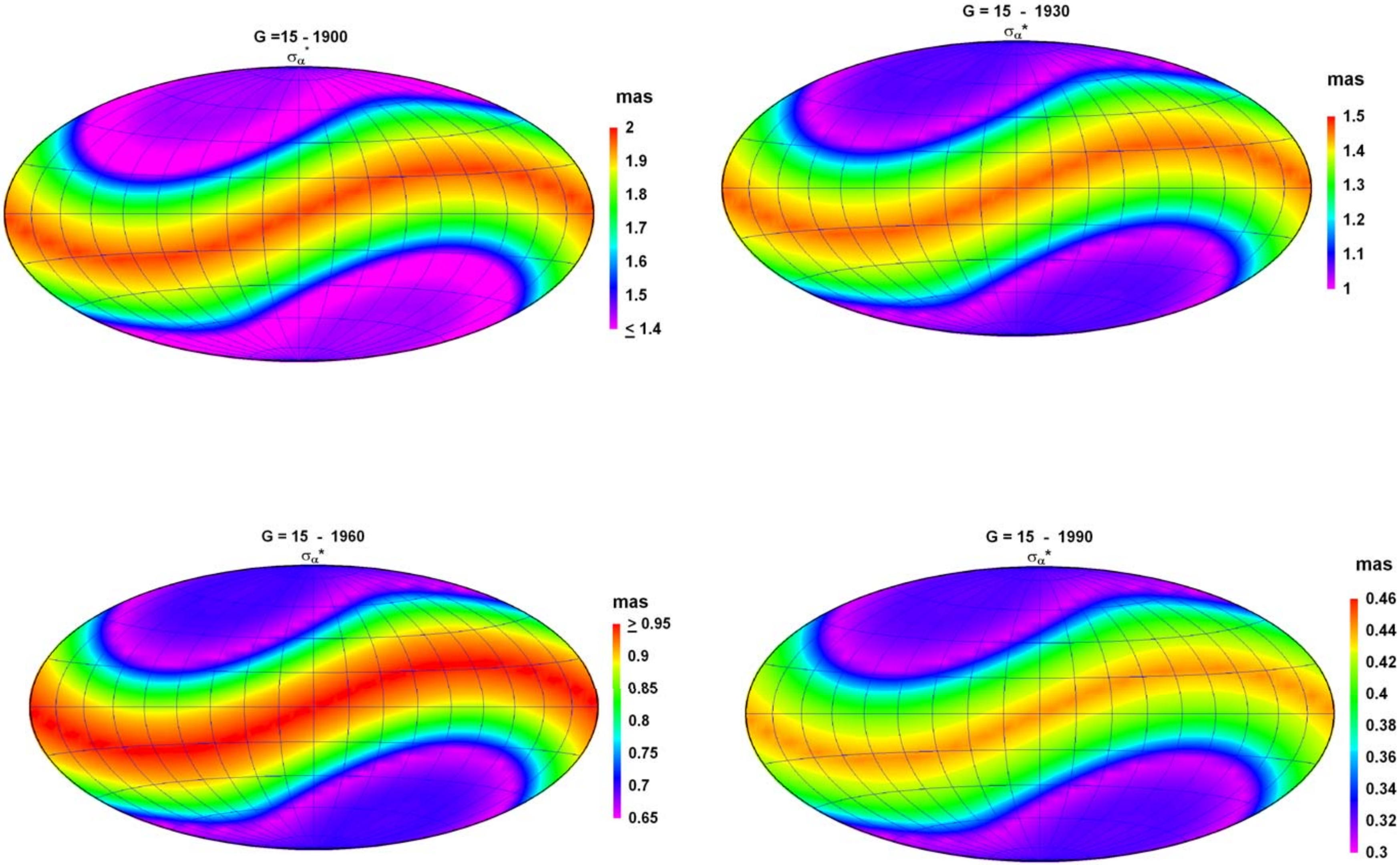




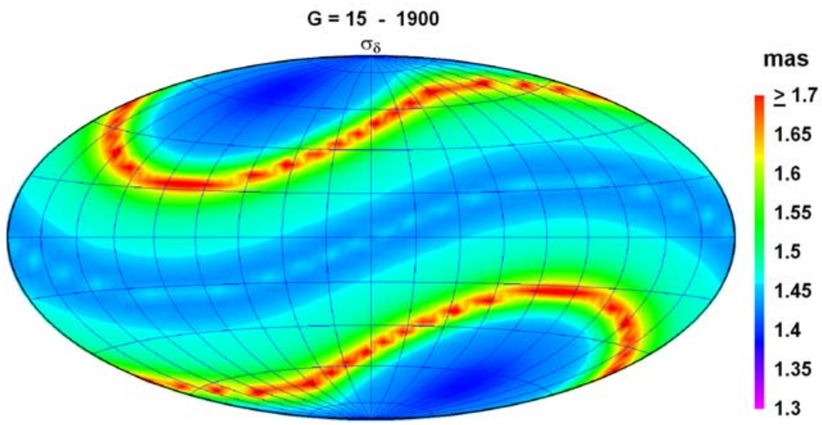
σ_δ 1900-1990 - G = 12

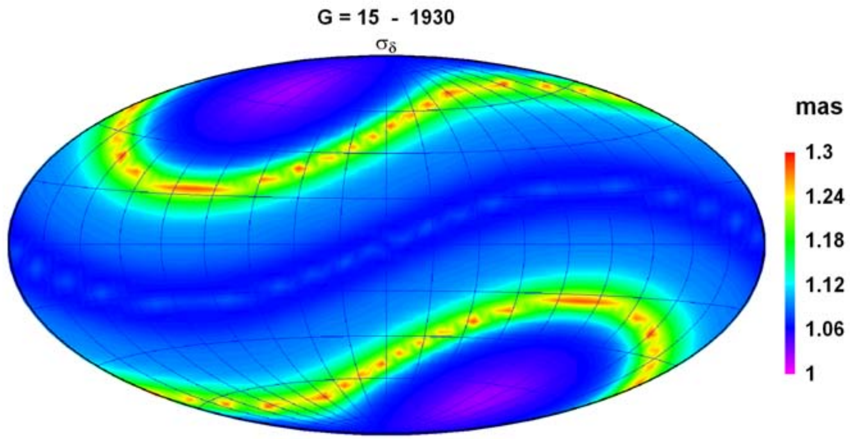


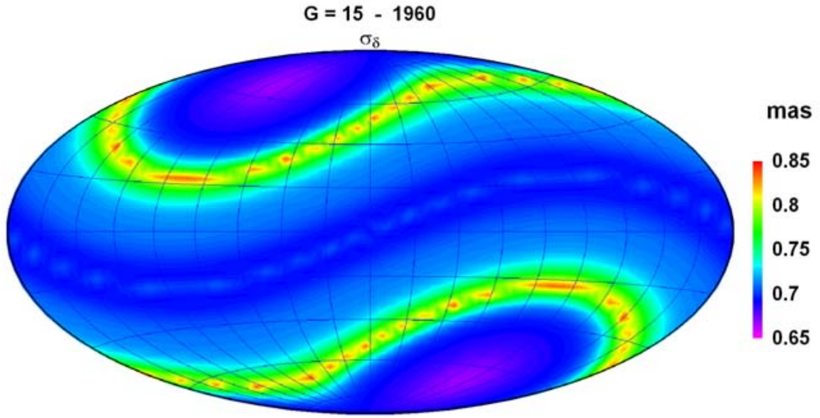
σ_α 1900-1990 - G = 15

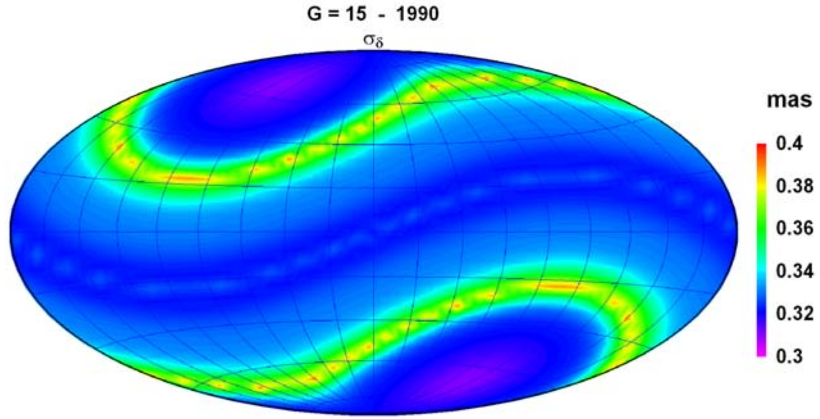


σ_δ 1900-1990 - G = 15









Radial velocity issue

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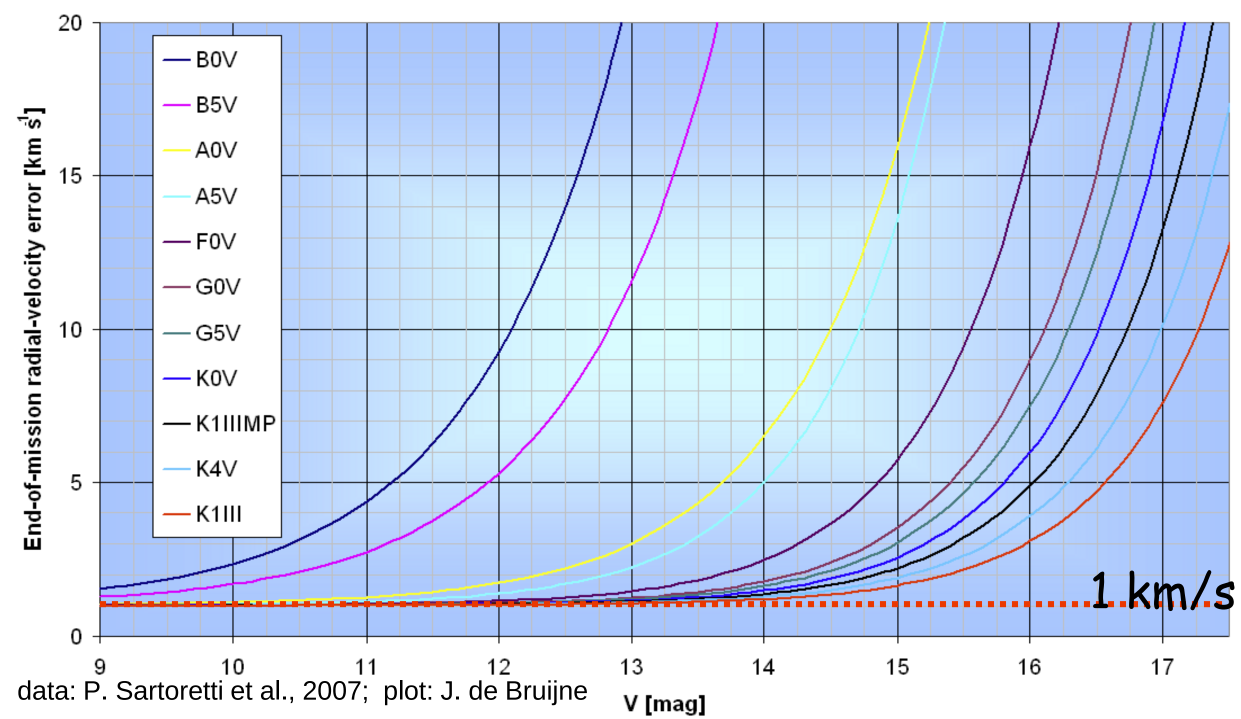
Radial Velocity

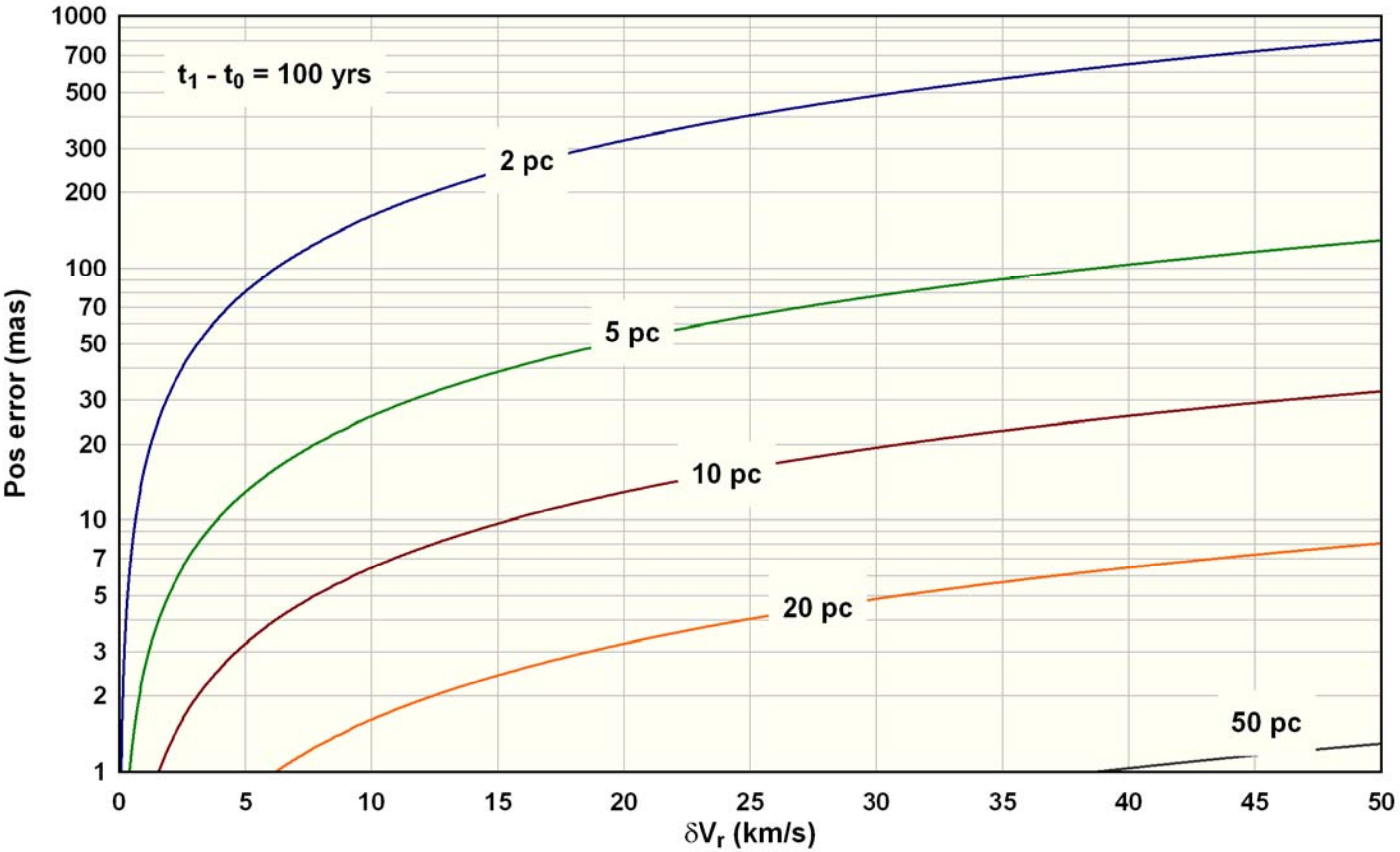
- To propagate without model truncation one needs the Vr
  - really needed for the nearby stars
- This will be available to G = 15 -17 from Gaia
- Accuracy 1 to 5 km/s for G-K stars V = 15



data: P. Sartoretti et al., 2007; plot: J. de Bruijne

Modelling errors

- If the error in V_r is $\delta(V_r)$ then there is an error in the computed position of the reference stars



Conclusions

- The potential of the Gaia catalogue to re-analyse old observations is unique
- The accuracy in the range V =14-16 is 1-3 mas back in 1900
- Given this unusual accuracy, care must be exercised in the astrometric modelling
  - linear propagation almost always too crude
- Radial velocities may become important for nearby stars
  - d < 20 pc

1. Thank you for you attention